

**IA Mathematics for Natural Sciences:  
'Easy A-level questions' - past paper questions**

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**2025, Paper 1, Section A, Question 1**

Solve for real  $x$ ,  $4^x - 2^x - 2 = 0$ . [2]

**2025, Paper 1, Section A, Question 2**

Given that the perimeter and area of a regular  $n$ -side polygon inscribed in a unit circle are equal to  $mn \sin(\frac{\pi}{n})$  and  $\frac{n}{k} \sin(\frac{2\pi}{n})$ , respectively, where  $m$  and  $k$  are integers, find  $m$  and  $k$ . [2]

**2025, Paper 1, Section A, Question 6**

Evaluate:

(a)  $-\frac{1}{2} + \frac{3}{2} - \frac{5}{2} + \frac{7}{2} - \dots + \frac{95}{2} - \frac{97}{2} + \frac{99}{2}$ , [1]

(b)  $\sum_{n=3}^{\infty} \left(\frac{1}{3}\right)^n$ . [1]

**2025, Paper 1, Section A, Question 9**

Two circles are defined by the following equations:

$$(x - 1)^2 + (y - 1)^2 = 1,$$

$$(x + 1)^2 + (y - 1)^2 = 1.$$

Find:

(a) the coordinates of a point belonging to both circles, [1]

(b) the distance between the centres of the circles. [1]

**2024, Paper 1, Section A, Question 4**

(a) Calculate the sum of the odd integers from 5 to 55 inclusive. [1]

(b) Calculate the sum of the infinite geometric series with first term 4 and common ratio  $\frac{1}{3}$ . [1]

**2024, Paper 1, Section A, Question 5**

Solve the following inequality for  $x$ :  $x \geq (x - 1)^2$ . [2]

**2024, Paper 1, Section A, Question 6**

(a) How many real solutions exist for the following simultaneous equations? [1]

$$\begin{cases} x^2 + y^2 = 1, \\ y = (x - 1)^2. \end{cases}$$

(b) What are they? [1]

**2024, Paper 1, Section A, Question 7**

Simplify the following equation:

$$2 \sec(x) - \tan(x) = \sqrt{3}$$

and then solve it for  $x$  in the range  $0 \leq x < \pi/2$ .

[2]

**2024, Paper 1, Section A, Question 8**

Find the radius and the coordinates of the centre of the following circle:

[2]

$$4x^2 - 8x + 4y^2 + 4y = 11.$$

**2023, Paper 1, Section A, Question 1**

Determine the numerical values of the coefficients of  $x^4$  and  $x^5$  in the expansion of  $(1 + x)^8$ .

[2]

**2023, Paper 1, Section A, Question 2**

Given that  $x = 3/2$  is a root of the cubic polynomial

$$2x^3 - 5x^2 + x + 3,$$

find the other two roots.

[2]

**2023, Paper 1, Section A, Question 3**

Find all solutions for  $x$  and  $y$  of the simultaneous equations:

[2]

$$\begin{cases} x + y = 1, \\ x - x^2 + 2y^2 = 2. \end{cases}$$

**2023, Paper 1, Section A, Question 4**

Find the radius and the coordinates of the centre of the circle  $x^2 + y^2 + 2x + 4y = 4$ .

[4]

**2023, Paper 1, Section A, Question 5**

Find the values of the parameters  $A$  and  $\phi$ , with  $A > 0$  and  $0 \leq \phi < 2\pi$ , such that:

$$\cos(t) + \sqrt{3} \sin(t) = A \sin(t + \phi),$$

for all real values of  $t$ .

[2]

**2023, Paper 1, Section A, Question 6**

Find integers  $p$  and  $q$  such that:

[2]

$$\sum_{n=1}^{10} 2^{-(n/2)} = \frac{p + q\sqrt{2}}{32}.$$

**2022, Paper 1, Section A, Question 1**

(a) Factorise the expression  $a^3 - b^3$  in terms of real-valued factors.

[1]

(b) Simplify the expression  $(a^{1/3} + b^{1/3})[a^{2/3} - (ab)^{1/3} + b^{2/3}]$ .

[1]

**2022, Paper 1, Section A, Question 2**

Solve for real  $x$ :  $\sqrt{x^2 - 2x + 1} = 2$ . [2]

**2022, Paper 1, Section A, Question 3**

Solve the inequality  $1/x \geq -1$ . [2]

**2022, Paper 1, Section A, Question 4**

Solve the set of simultaneous equations for real  $x$  and  $y$ : [2]

$$\begin{cases} 3^x + 3^y = 2, \\ 3^{x+y} = 1. \end{cases}$$

**2022, Paper 1, Section A, Question 5**

Evaluate:

(a)  $-2 + 4 - 6 + 8 - 10 + \dots - 98 + 100$ , [1]

(b)  $\sin(\pi/4) - \sin^2(\pi/4) + \sin^3(\pi/4) - \sin^4(\pi/4) + \dots$ . [1]

**2022, Paper 1, Section A, Question 9**

Find the centre and the radius of the circle  $x^2 - 4x + y^2 + 3 = 0$ . [2]

**2021, Paper 1, Section A, Question 2**

Factorise  $x^2 - x - 2$  and hence find all  $x$  for which  $x^2 - x - 2 < 0$ . [2]

**2021, Paper 1, Section A, Question 3**

Find all  $\theta$  in the interval  $0 \leq \theta \leq \pi$  such that: [2]

$$\sin^4(\theta) + \cos^4(\theta) = \frac{1}{2}.$$

**2021, Paper 1, Section A, Question 4**

Calculate the values of  $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$  and  $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \dots$ . [2]

**2021, Paper 1, Section A, Question 9**

Given  $t = \tan(\theta/2)$ , give expressions for  $\tan(\theta)$  and  $\sec^2(\theta)$  in terms of  $t$ . [2]

**2020, Section A, Question 1**

Given that  $x - 2$  is a factor of  $x^3 - x^2 - x - 2$ , find the other factor. [2]

**2020, Section A, Question 2**

Solve for  $\theta$  in the interval  $\pi/2 < \theta < 3\pi/2$ : [2]

$$(\sin(\theta) - \cos(\theta))(\sin^2(\theta) + 2\cos^2(\theta)) \leq 0.$$

**2020, Section A, Question 4**

Consider the curves  $x^2 + 2y^2 = 1$  and  $ax^2 - y^2 = 1$  where  $x, y$  and the constant  $a$  are real, and  $a > 0$ . For what values of  $a$  do the curves have no shared points? [2]

**2020, Section A, Question 5**

Given  $\tan(\pi/4) = 1$ ,  $\tan(\pi/8) = t$ , and using the formula for  $\tan(2\theta)$  in terms of  $\tan(\theta)$ :

(a) Find a quadratic equation for  $t$ . [1]

(b) Solve this equation to find  $\tan(\pi/8)$ . [1]

**2020, Section A, Question 9**

Write down an explicit formula for the sum to one hundred terms of the geometric series with common ratio  $r$  and first term  $ar$ . For what values of  $a$  and  $r$  is your formula valid? [2]

**2019, Paper 1, Section A, Question 1**

Solve for  $x$ :  $2x^2 + x \leq 6$ .

**2019, Paper 1, Section A, Question 2**

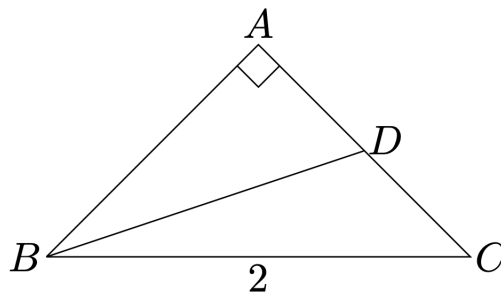
Find the coordinates of the centre of the ellipse:  $x^2 - 4x + 4y^2 - 8y = -4$ . [2]

**2019, Paper 1, Section A, Question 4**

The diagram below shows an isosceles triangle  $ABC$  with a right angle at  $A$ .  $D$  is the midpoint of  $AC$ . The length of  $BC$  is 2 units. Calculate:

(a) The length  $AD$ ; [1]

(b) The cosine of the angle  $ADB$ . [1]

**2019, Paper 1, Section A, Question 5**

Solve for  $x$  in the interval  $0 \leq x \leq \pi$ :  $\cos(2x) = \sin(x)$ . [2]

**2019, Paper 1, Section A, Question 7**

Evaluate:

(a)  $\sum_{n=2}^{101} 2n;$  [1]

(b)  $\sum_{n=1}^5 4^n.$  [1]

**2018, Paper 1, Section A, Question 1**

(a) Solve the equation  $2^x = 10^y$ , for  $x$ . [1]

(b) Calculate  $x$  to two significant figures for the case  $y = 23.8$ , given  $\log_{10}(2) \approx 0.301$ . [1]

**2018, Paper 1, Section A, Question 2**Solve for  $x$ :

(a)  $2x^3 + 9x^2 - 6x = 5,$  [1]

(b)  $x^2 + 3x + 1 < 0.$  [1]

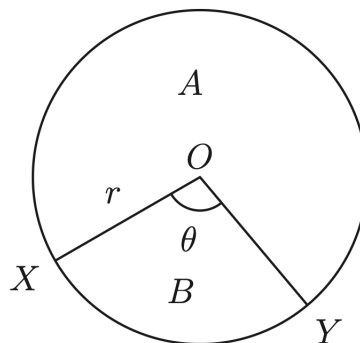
**2018, Paper 1, Section A, Question 3**Calculate the values of  $t$  and  $\phi$  for the points of intersection of the two curves: [2]

$$\begin{aligned} x &= 2 \cos(\phi), & y &= \sin(\phi); & 0 &\leq \phi < 2\pi, \\ \text{and} & & x &= -2t, & y &= 1 + t; & -\infty < t < \infty. \end{aligned}$$

**2018, Paper 1, Section A, Question 5** $O$  is the centre of a circle of radius  $r$  as shown in the diagram.

(a) Given that the area  $A$  is twice the area  $B$ , calculate  $\theta$ . [1]

(b) In terms of the area  $B$ , calculate the area of the triangle  $OXY$ . [1]



**2018, Paper 1, Section A, Question 7**

- (a) Calculate to three decimal places  $(1.01)^9$ . [1]
- (b) Calculate  $\sum_{n=-1}^{\infty} \left(\frac{1}{3}\right)^n$ . [1]

**2018, Paper 1, Section A, Question 9**

Consider the circle defined by the equation  $x^2 + y^2 + x + 2y = 1$ .

- (a) Find the coordinates of its centre. [1]
- (b) Find its radius. [1]

**2017, Paper 1, Section A, Question 5**

- (a) Verify that the polynomial  $2x^3 + 3x^2 - 11x - 6$  changes sign between  $x = 0$  and  $x = 3$ , and so find one of its roots by trial. [1]
- (b) Hence factorise the polynomial. [1]

**2017, Paper 1, Section A, Question 6**

- (a) Write the polynomial  $1 + 8x + 24x^2 + 32x^3 + 16x^4$  in the form  $(a + bx)^m$ . [1]
- (b) Hence sketch the graph of the polynomial in the range  $-1 \leq x \leq 1$  and calculate the points of intersection with the  $x$  and  $y$  axes. [1]

**2017, Paper 1, Section A, Question 7**

- (a) Write  $\sin^2(\pi/12)$  in terms of  $\sin(\pi/6)$ . [1]
- (b) Given  $\sqrt{3} \approx 1.732$ , calculate  $\sin^2(\pi/12)$  to two decimal places. [1]

**2017, Paper 1, Section A, Question 8**

- (a) Solve  $\cos(x) = \sin(2x)$  for  $0 \leq x \leq \pi/2$ . [1]
- (b) Solve  $1 + \cos(2x) = \cos^2(x)$  for  $\pi < x < 2\pi$ . [1]

**2016, Paper 1, Section A, Question 1**

- (a) Factorise the expression  $a^3 + b^3$ . [1]
- (b) Find the repeated root of the equation  $x^3 + x^2 - 16x + 20 = 0$ . [1]

**2016, Paper 1, Section A, Question 3**

Evaluate,

- (a)  $10002 \times 9998$ , [1]
- (b)  $\ln \left[ \frac{2}{\pi} (\arctan(\pi/4) + \arctan(4/\pi)) \right]$ . [1]

**2016, Paper 1, Section A, Question 4**

Find all real  $x$  obeying the inequality  $\sin(x) \leq -\sqrt{3}/2$ . [2]

**2016, Paper 1, Section A, Question 5**

(a) Sketch  $|y| = 1 - x^2$  in the  $xy$ -plane. [1]

(b) Find all the real solutions of the simultaneous equations,  $y = 1/2$  and  $|y| = 1 - x^2$ . [1]

**2016, Paper 1, Section A, Question 10**

Find graphically, or otherwise, all the values of the real parameter  $k$  for which the following set of simultaneous equations has real solutions for  $(x, y)$ , [2]

$$\begin{aligned}(x - 2)^2 + (y - 1)^2 &= 1, \\ y &= kx.\end{aligned}$$

**2015, Paper 1, Section A, Question 8**

Solve the following equation for  $x$  in the range  $-\pi < x < \pi$ , with  $x \neq 0$ :  $\cot(x) = \sin(2x)$ . [2]

**2015, Paper 1, Section A, Question 10**

Evaluate:

(a)  $N_1 = 1 + 2 + 3 + \cdots + 999 + 1000$ , [1]

(b)  $N_2 = 2 + 4 + 8 + \cdots + 1024 + 2048$ . [1]

**2014, Paper 1, Section A, Question 5**

(a) Find all pairs of coordinates where the curve defined by  $7x^2 - y^2 = 7$  meets the straight line  $y = x + 1$ . [1]

(b) Sketch the curve defined by the equation  $(x - 1)^2 + 2y^2 = 3$ . [1]

**2014, Paper 1, Section A, Question 6**

(a) A sphere has radius 10 m. Drawn on its surface is a circular patch which has area  $10 \text{ m}^2$ . What fraction of the surface of the sphere does this cover? [1]

(b) A square pyramid has all of its sides of length 1 m. What is its volume? [1]

**2012, Paper 1, Section A, Question 1**

Write  $f(x) = 3x^2 - 12x + 11$  in the form  $a(x + b)^2 + c$ . State the minimum value of  $f(x)$  and the value of  $x$  where this occurs. [2]

**2012, Paper 1, Section A, Question 3**

The function  $f(x)$  is given by  $f(x) = 2x^3 - px^2 - px + 21$ . When  $f(x)$  is divided by  $x - 1$  the remainder is 9. Find  $p$ . [1]

Divide  $x^3 + x^2 + 2x + 1$  by  $x - 1$  giving quotient and remainder. [1]

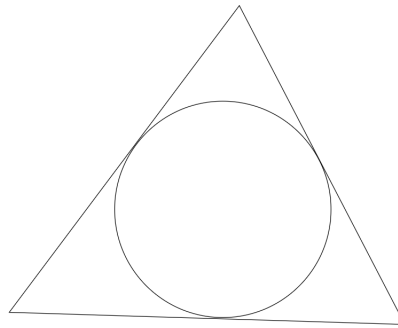
**2012, Paper 1, Section A, Question 8**

(a) Evaluate  $\sum_{n=4}^{\infty} \left(\frac{1}{2}\right)^n$ . [1]

(b) Calculate the term in  $x^3$  in the expansion of  $(1+x)^9$ . [1]

**2011, Paper 1, Section A, Question 5**

A circle is inscribed within an equilateral triangle as shown below. Find the ratio of the area of the circle to the area of the triangle.

**2011, Paper 1, Section A, Question 6**

Given that  $\theta = \pi/2$  is a solution of:

$$\sin^3(\theta) - \frac{1}{4}\sin(\theta) = \sin^2(\theta) - \frac{1}{4},$$

find all the other solutions for  $\theta$  explicitly, in the range  $-\pi \leq \theta \leq \pi$ . [2]

**2011, Paper 1, Section A, Question 10**

Evaluate:

(a)  $\sum_{n=-100}^{99} (n+1)^3$ , [1]

(b)  $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{3k}$ . [1]

**2010, Paper 1, Section A, Question 1**

Given that  $x = 1$  is a solution of  $2x^3 - 9x^2 + 13x - 6 = 0$ , find all solutions of this equation. [2]

**2010, Paper 1, Section A, Question 5**

Determine the radius and the position of the centre of the circle  $x^2 + y^2 - 6x - 2y + 1 = 0$ . [2]

**2010, Paper 1, Section A, Question 7**

Evaluate:

$$(a) \sum_{n=0}^{\infty} (0.999)^n, \quad [1]$$

$$(b) \sum_{n=0}^N (n+1). \quad [1]$$

**2010, Paper 1, Section A, Question 9**

$$\text{Find the solutions of } \sqrt{2x^2 + 17} + x = 3. \quad [2]$$

**2010, Paper 1, Section A, Question 10**

$$\text{Solve } 1 + \sin(2x) = 2 \cos^2(x) \text{ for } 0 \leq x \leq \pi. \quad [2]$$

**2009, Paper 1, Section A, Question 1**

$$\text{Given that } x = 3 \text{ is a solution of } x^3 - x^2 - 8x + 6 = 0 \text{ find all the other solutions of this equation.} \quad [2]$$

**2009, Paper 1, Section A, Question 2**

$$\text{Find the solution of the equation: } \sqrt{x+16} - x + 4 = 0. \quad [2]$$

**2009, Paper 1, Section A, Question 5**

$$\text{Find all solutions of the equation } 1 + \cos(x) + \cos(2x) = 0 \text{ in the range } 0 \leq x \leq \pi. \quad [2]$$

**2009, Paper 1, Section A, Question 6**

$$\text{Find the solution of the equation in the simplest form:} \quad [2]$$

$$\frac{\log_{10}(x)}{1 - \log_{10}(2)} = 2.$$

**2009, Paper 1, Section A, Question 7**

Evaluate:

$$(a) \sum_{n=0}^{100} (3n+2); \quad [1]$$

$$(b) \sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n. \quad [1]$$

**2009, Paper 2, Section A, Question 9**

$$\text{Find the values of } \theta \text{ in } [0, 2\pi] \text{ for which } |\sqrt{2} \cos(\theta)| > 1. \quad [2]$$

**2008, Paper 1, Section A, Question 1**

$$\text{Find all solutions of the equation } \sqrt{3} \cos(\theta) + \sin(\theta) = \sqrt{2} \text{ in the range } 0 \leq \theta < 2\pi. \quad [2]$$

**2008, Paper 1, Section A, Question 4**

Find all solutions of  $x^4 - 3x^2 + 2 = 0$ . [2]

**2008, Paper 1, Section A, Question 5**

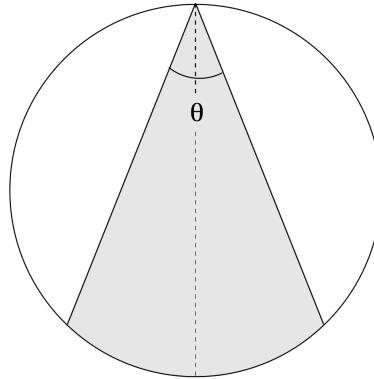
Find the distance between the intersection of the curves  $xy = 1$  and  $x = 2y$ . [2]

**2008, Paper 1, Section A, Question 8**

Find the ranges of  $x$  for which the inequality  $|x - 2| + |x - 1| > 1$  is satisfied. [2]

**2008, Paper 1, Section A, Question 9**

In the figure below, the shaded area is divided equally by the diameter of a circle of unit radius. Find the shaded area in terms of the angle  $\theta$ . [2]

**2008, Paper 1, Section A, Question 10**

(i) Evaluate  $\sum_{n=0}^N (1+n)^3 - \sum_{n=0}^N n^3$ . [1]

(ii) Hence, or otherwise, evaluate  $\sum_{n=0}^N n(1+n)$ . [1]

**2007, Paper 1, Section A, Question 1**

Given that  $x = 1$  is a solution of  $2x^3 + x^2 - 5x + 2 = 0$ , find all solutions of this equation. [2]

**2007, Paper 1, Section A, Question 4**

Determine the radius and position of the centre of the circle  $x^2 + y^2 + 6y + 5 = 0$ . [2]

**2007, Paper 1, Section A, Question 5**

(a)  $\sum_{n=0}^{100} (2n + 1);$  [1]

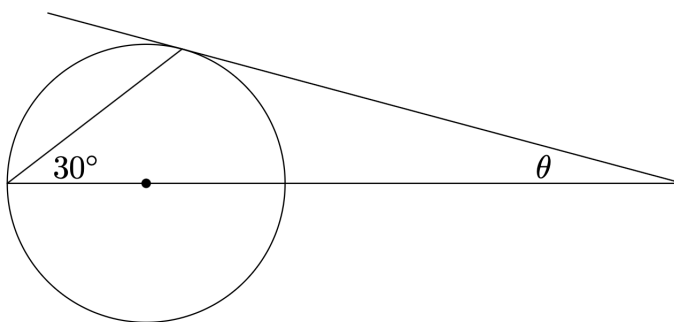
(b)  $\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n.$  [1]

**2007, Paper 1, Section A, Question 7**

Find all solutions of the equation  $\sin(2\theta) - 2\cos^2(\theta) = 0$  in the range  $0 \leq \theta \leq \pi$ . [2]

**2007, Paper 1, Section A, Question 8**

Determine the angle  $\theta$ . The picture is not drawn accurately.

**2007, Paper 1, Section A, Question 9**

Write  $\sqrt{5} \left( \frac{\sqrt{5} + 1}{2} + \frac{2}{\sqrt{5} + 1} \right)$  in its simplest form. [1]