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EFFICIENT SIMULTANEOUS DETERMINATION OF PDFS & SMEFT COUPLINGS

SUMMARY OF PRESENTATION

1. Brief review of problem: Simultaneous determination of PDFs and SMEFT couplings
2. Existing determinations: 'Can New Physics Hide Inside the Proton?' and 'Parton Distributions in the SMEFT from High Energy DY Tails'
3. The Weight-Minimisation Methodology - originally Zahari's idea, also worked on by Shayan, Maria and myself
4. Current Results from the Weight-Minimisation Methodology

SECTION 1

REVIEW OF PROBLEM: SIMULTANEOUS EXTRACTION OF PDFS AND SMEFT COUPLINGS

BRIEF REVIEW OF PROBLEM

- BSM theorists often make use of the **Standard Model EFT**: append all possible higher-dimension (non-renormalisable) operators, consistent with SM symmetries and built from SM fields, to SM Lagrangian

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{i=1}^{\#\text{ dim 6 ops}} \mathcal{O}_i^{(6)} + \sum_{i=1}^{\#\text{ dim 8 ops}} \mathcal{O}_i^{(8)} + \cdots$$

- **Idea:** integrating out fields in a UV-complete theory gives non-renormalisable operators, so this accounts for all possible SM extensions (with important assumptions) - the SMEFT is '**unbiased'** (up to these assumptions)

BRIEF REVIEW OF PROBLEM

- In SMEFT studies, the goal is to place bounds on the SMEFT couplings, called **Wilson coefficients**
- Achieved by minimising χ^2-statistic of predictions to data over the space of SMEFT couplings
- **Important issue:** Predictions always made with SM PDFs!

$$\sigma_{\mathrm{BSM}} = \hat{\sigma}_{\mathrm{BSM}} \otimes f_{\mathrm{SM}}$$

SM PDFs were fitted assuming SM!

BRIEF REVIEW OF PROBLEM

- Since the data used in SMEFT fits overlaps with data used in PDF fits, this is an **inconsistent approach -** but is it a problem? Need a robust response to the questions:

To what extent do bounds on SMEFT couplings change when PDFs are fitted simultaneously alongside the SMEFT couplings?

Can we make sure that our PDF sets are not contaminated by BSM effects (e.g. by using 'conservative' PDF sets)?

- Very important: inconsistent PDF treatment could result in **missing New Physics** or **seeing New Physics that isn't there**

SECTION 2

EXISTING SIMULTANEOUS PDF-SMEFT DETERMINATIONS

SUMMARY OF EXISTING SIMULTANEOUS SMEFT-PDF DETERMINATIONS

- To date, two important papers on the simultaneous determination of PDFs and SMEFT couplings

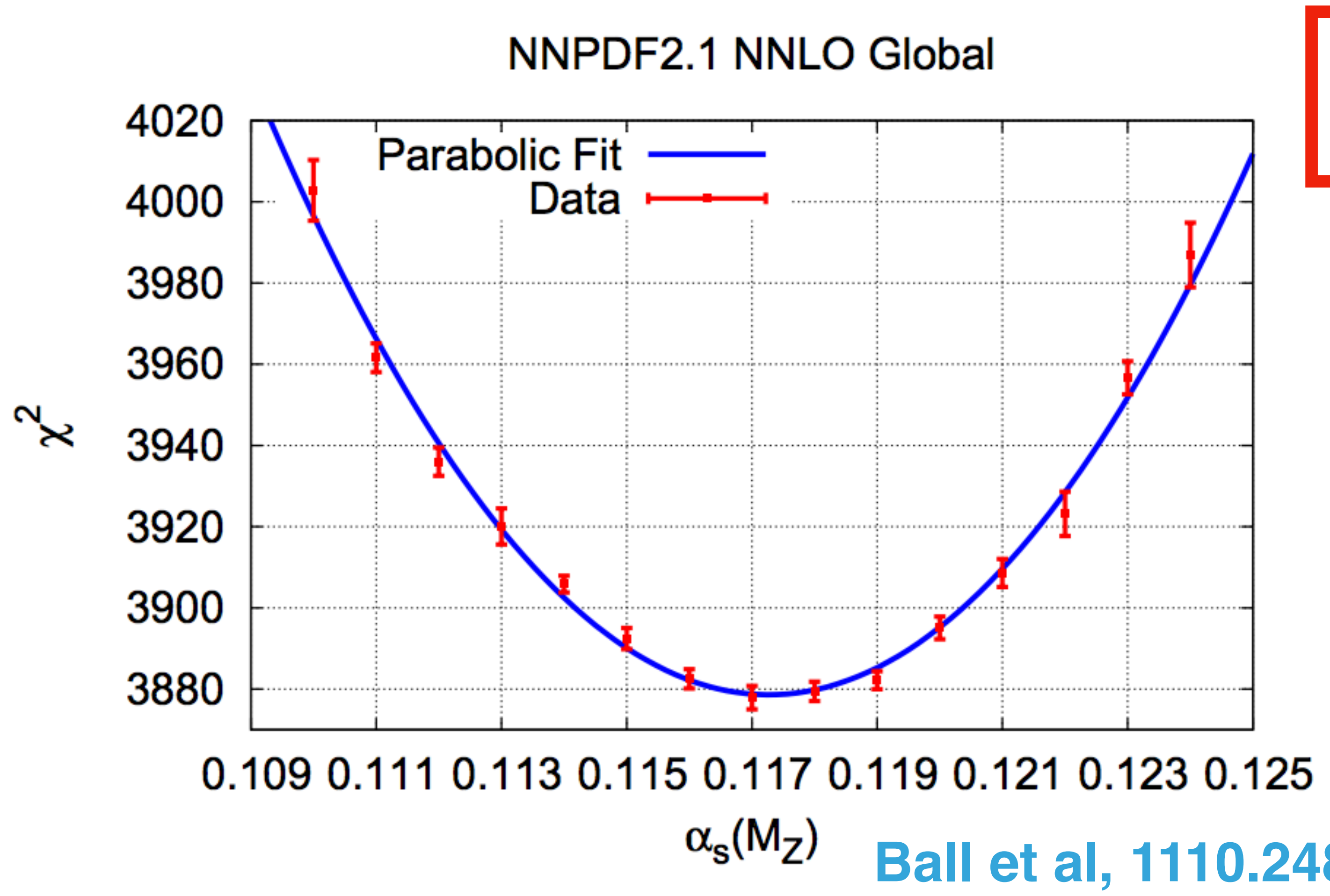
- **'Can New Physics Hide Inside the Proton?'** - 2019, Carrazza, Degrande, Iranipour, Rojo, Ubiali

- **'Parton distributions in the SMEFT from high-energy Drell-Yan tails'** - 2021, Greljo, Iranipour, Kassabov, Madigan, Moore, Rojo, Ubiali, Voisey

- Both studies use same methodology

OLD METHODOLOGY (slide modified from Maria's)

- The method used for the DIS and DY papers was the same as the 'old α_S' method
- We **don't** use the **correlated replica** method, as this is more computationally expensive and the PDF-SMEFT correlation is weaker than the PDF-α_S correlation

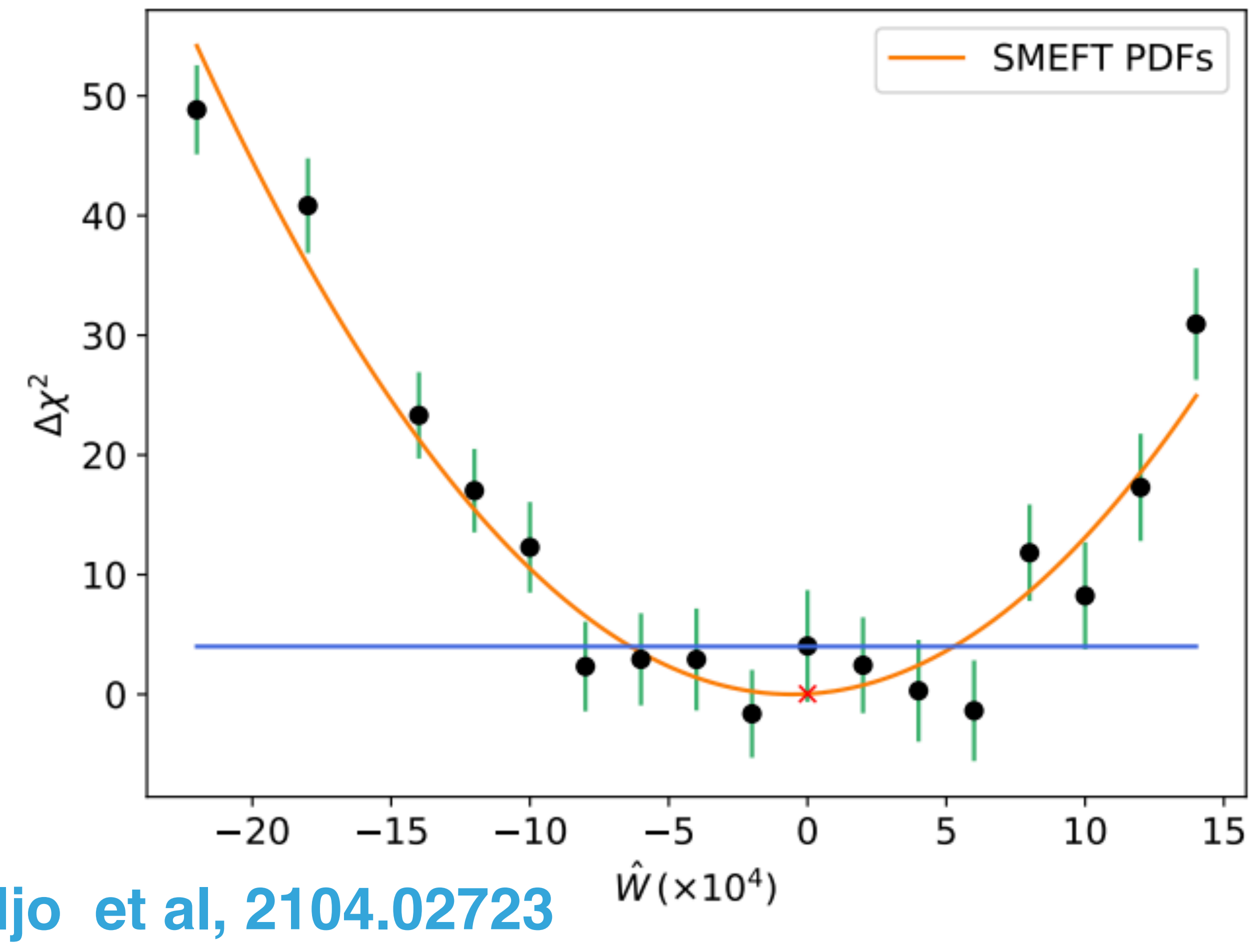


Ball et al, 1110.2483

$$T = f_1(\alpha_s) \otimes f_2(\alpha_s) \otimes \hat{\sigma}(\alpha_S)$$

$$T = f_1(\hat{W}) \otimes f_2(\hat{W}) \otimes \hat{\sigma}(\hat{W})$$

$$\chi^2 = \frac{1}{n_{\rm dat}} \sum_{i,j=1}^{n_{\rm dat}} (D_i - T_i)\,({\rm cov}^{-1})_{ij}\,(D_j - T_j)$$



Greljo et al, 2104.02723

SM PDFs

1. Take data, make theoretical predictions accounting for operator in partonic cross section with fixed SM PDFs.
2. Compute chi2 as a function of SMEFT couplings
3. Minimise chi2 and find best-fit and C.L.s of SMEFT couplings
4. Extract bounds

$$T = f_{1,\rm SM} \otimes f_{2,\rm SM} \otimes \hat{\sigma}_{\rm BSM}$$

SMEFT PDFs / Simultaneous fit

1. Take data, make theoretical predictions accounting for operator in partonic cross section and PDFs.
2. Compute chi2 as a function of SMEFT couplings
3. Minimise chi2 and find best-fit and C.L.s of SMEFT couplings
4. Extract bounds

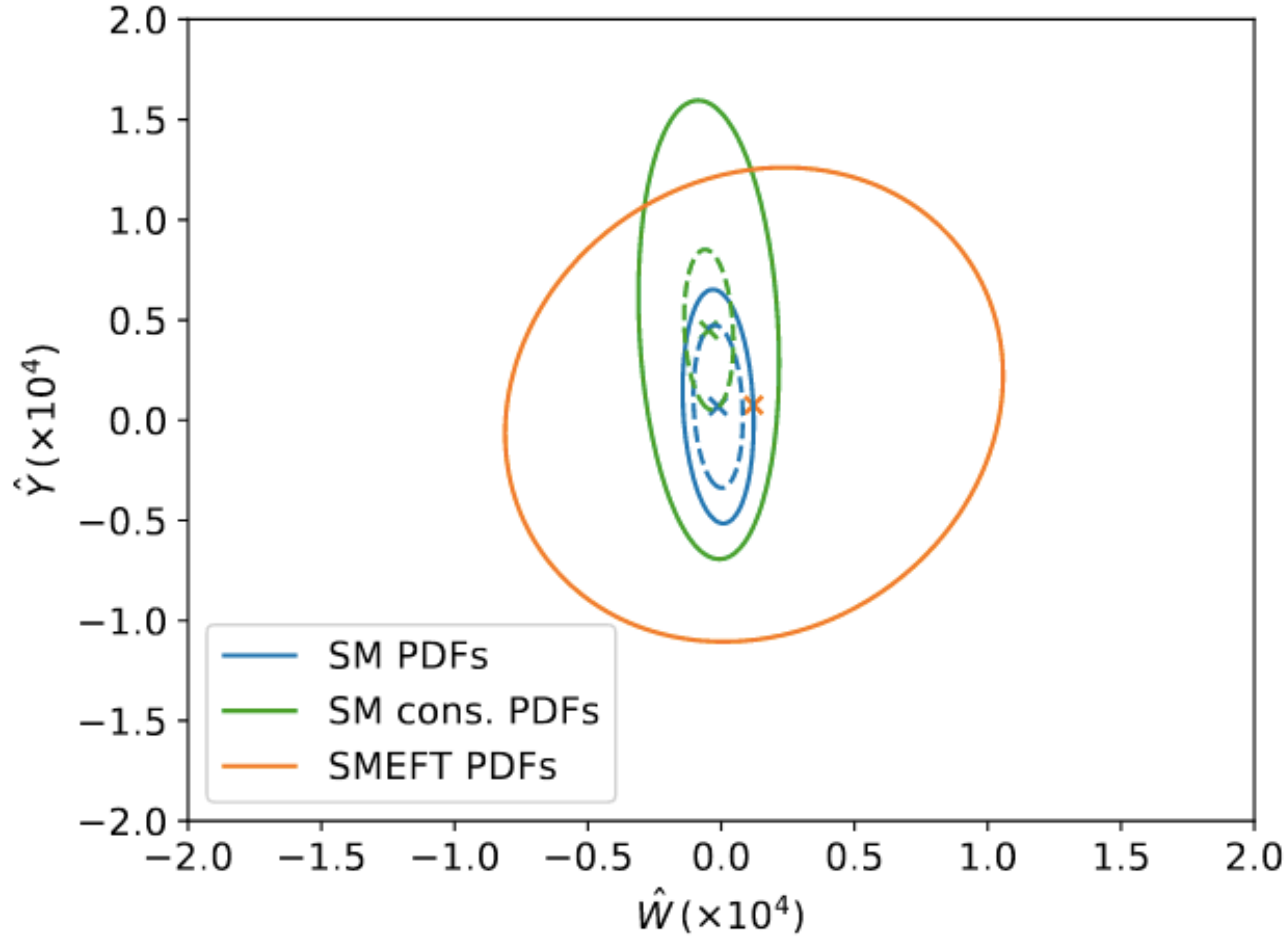
$$T = f_{1,\rm BSM} \otimes f_{2,\rm BSM} \otimes \hat{\sigma}_{\rm BSM}$$

CURRENT METHODOLOGY

- Mixture of Monte Carlo and Hessian approaches
- Uses **partial** χ^2 (computed only on the SMEFT-modified data) rather than **global** χ^2
- **Key problem:** methodology becomes exponentially slower as number of SMEFT couplings increases - inappropriate for scenarios with even a moderate number (>4, say).
- **Essential for the future that we have a more efficient methodology.**

SUMMARY OF EXISTING SIMULTANEOUS SMEFT-PDF DETERMINATIONS

- 2021 paper showed that consistent PDF treatment is **already necessary** with the current high-mass DY data, and will become **vital** for future high-luminosity LHC data



SECTION 3

THE WEIGHT-MINIMISATION METHODOLOGY

LINEARISATION

- Idea of weight-minimisation methodology is to take simplest possible approach: attempt to fit the difference between **existing SM PDFs** and **SMEFT PDFs** using a **linear model**

Replica number

$$f^{(j)}_{\text{SMEFT}}(x, Q^2) - f^{(j)}_{\text{SM}}(x, Q^2) = \sum_{i=1}^{N} w_i^{(j)} h_i(x, Q^2)$$

'Weights'

Some basis functions

LINEARISATION

- The basis functions $h_i(x, Q^2)$ are constrained by some theory requirements
- **Linearity of DGLAP equations** implies $h_i(x, Q^2)$ must solve DGLAP equations
- **PDF sum rules** (valence sum rules + momentum sum rule) imply that $h_i(x, Q^2)$ obey homogeneous versions of the sum rules
- Theory constraints are satisfied by choosing $h_i(x, Q^2)$ to be **differences of existing PDF replicas**

LINEARISATION

- Reasonable choice satisfying all conditions:

$$f^{(j)}_{\text{SMEFT}}(x, Q^2) - f^{(j)}_{\text{SM}}(x, Q^2) = \sum_{i=1}^{N} w_i^{(j)} \left(f^{(i)}_{\text{SM}}(x, Q^2) - f^{(j)}_{\text{SM}}(x, Q^2) \right)$$

Idea: ‘expansion in directions of other PDF replicas’

LINEARISATION

- Using this expansion, theory predictions can be linearised in terms of both **weights** and **SMEFT couplings**
- E.g. for DIS predictions:

$$\hat{\sigma}_{\text{SMEFT}} \otimes f^{(j)}_{\text{SMEFT}} = \left(\hat{\sigma}_{\text{SM}} + \sum_{i=1}^{\#\,\text{ops}} a_i^{(j)} \hat{\sigma}_i \right) \otimes \left(f^{(j)}_{\text{SM}} + \sum_{i=1}^{N} w_i^{(j)} \left(f^{(j)}_{\text{SM}} - f^{(i)}_{\text{SM}} \right) \right)$$

$$= \hat{\sigma}_{\text{SM}} \otimes f^{(j)}_{\text{SM}} + \sum_{i=1}^{N} w_i^{(j)} \hat{\sigma}_{\text{SM}} \otimes \left(f^{(j)}_{\text{SM}} - f^{(i)}_{\text{SM}} \right) + \sum_{i=1}^{\#\text{ops}} a_i^{(j)} \hat{\sigma}_i f^{(j)}_{\text{SM}} + O(a \cdot \Delta f)$$

Discarded as expected to be small

LINEARISATION

- Similar linearisation applies to more complicated DIS observables (e.g. ratio observables) with a little more work
- Similar linearisation applies to DY observables; terms that are quadratic in the weights are also discarded in this case
- Predictions can be packaged in matrix-vector form as:

$$\text{predictions} = \mathbf{T} + P\mathbf{w} + Q\mathbf{a}$$

- Clearly shows equal treatment of PDF deviation and SMEFT couplings

NAIVE APPROACH

- At this point, all predictions are linear, so χ^2 statistic (of each replica's predictions to each pseudodata replica) is quadratic form
- Minimum of χ^2 statistic can be found analytically
- Can also introduce **positivity** requirement on predictions, which makes problem a **convex optimisation problem** - can be solved with aid of Python library **cvxopt**

REGULARISATION & HYPEROPTIMISATION

- **Problem:** We have not yet said how many weights we should use in parametrising the PDF deviation - what's an appropriate choice?

- Too few: will **underfit** SMEFT PDF deviation from SM PDF
- Too many: will **overfit** SMEFT PDF deviation from SM PDF

REGULARISATION & HYPEROPTIMISATION

- We have seen that **it is indeed possible to overfit** - e.g. 100 weights is too many

- E.g. for above example, big χ^2 reduction: 1.162 → 1.108

REGULARISATION & HYPEROPTIMISATION

- **Solution:** use a **regularisation method** together with **hyperoptimisation**.

- We choose to use the L^2 **regularisation** method. This allows effective control over the space that the weights can explore (rather than limiting numbers of weights - though we have explored a 'drop-out' inspired approach where weights are omitted, and we see it is equivalent).

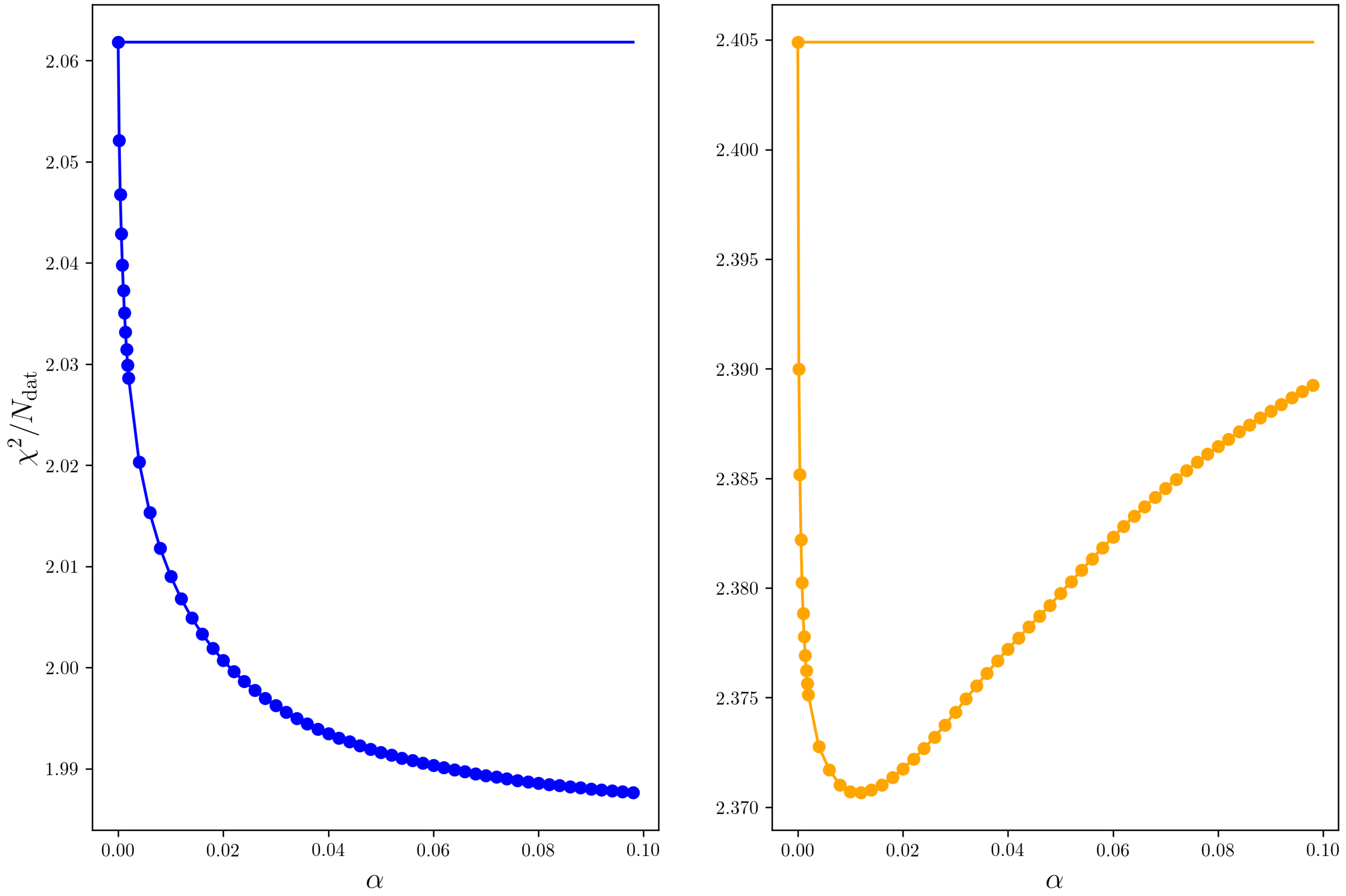
REGULARISATION & HYPEROPTIMISATION

- We fit the jth replica (together with SMEFT couplings in the SMEFT case) to the same **training pseudodata** that the jth replica in the original PDF fit saw
- The best-fit weights and SMEFT couplings are obtained by minimising the modified χ^2-statistic:

$$\chi^2_{\text{mod}}(\mathbf{w}, \mathbf{a}, \alpha) = \chi^2(\mathbf{w}, \mathbf{a}) + \frac{1}{\alpha} || \mathbf{w} ||^2$$

- The value of the hyperparameter α is chosen such that the weights and SMEFT couplings minimise the unmodified χ^2-statistic to the **validation pseudodata**

EXAMPLE TRAINING & VALIDATION CURVES



IMPORTANT BENCHMARKS

1. Methodology should **produce very similar bounds to previous two papers** in the fixed PDF case: only difference is Monte Carlo vs Hessian approach
2. Methodology should **approximately reproduce the baseline PDF set** when SMEFT couplings are fixed to zero
3. Methodology should be **able to detect New Physics if present**: can test by artificially adding New Physics to the pseudodata, and seeing if it can be accurately detected

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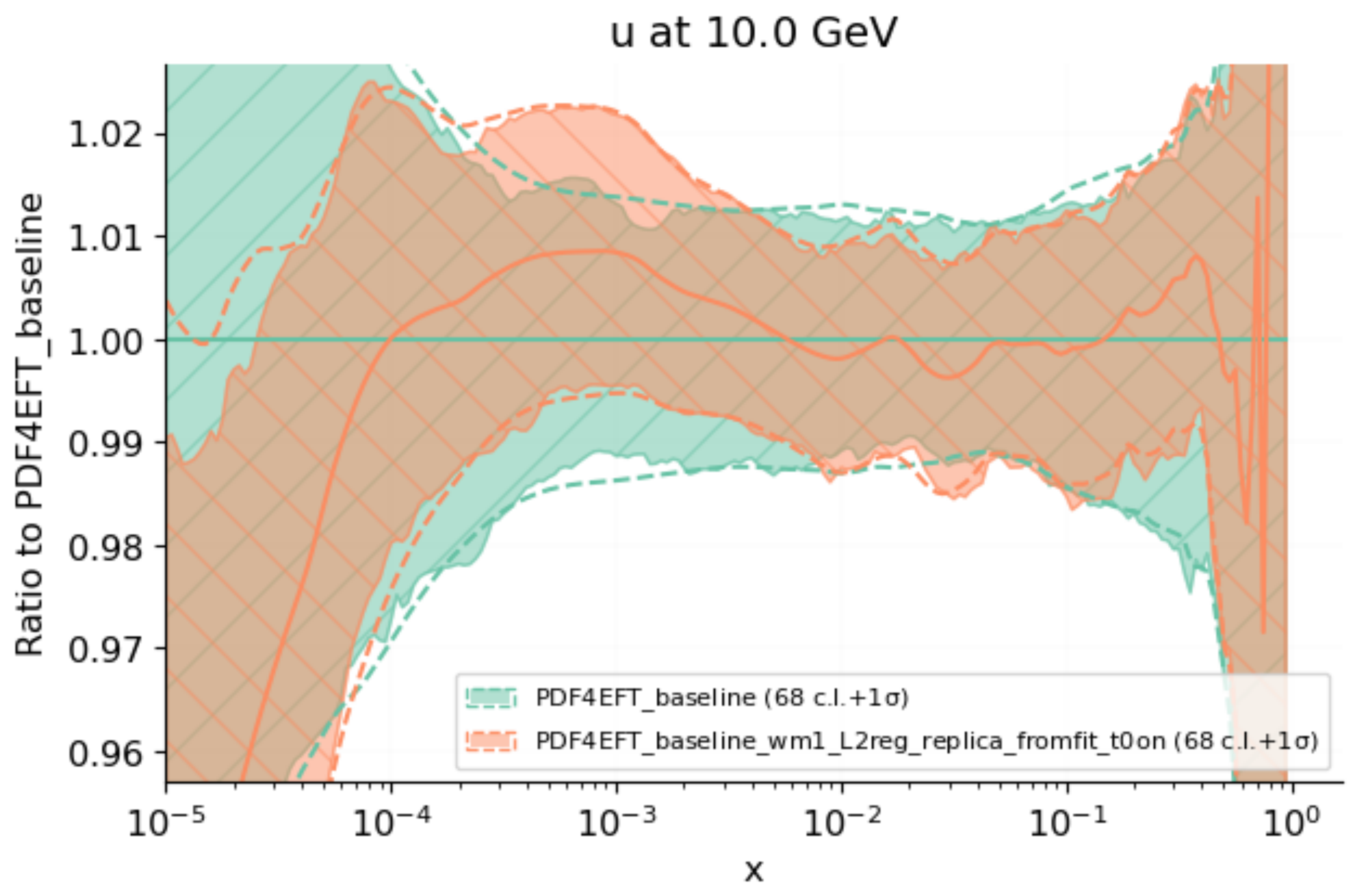
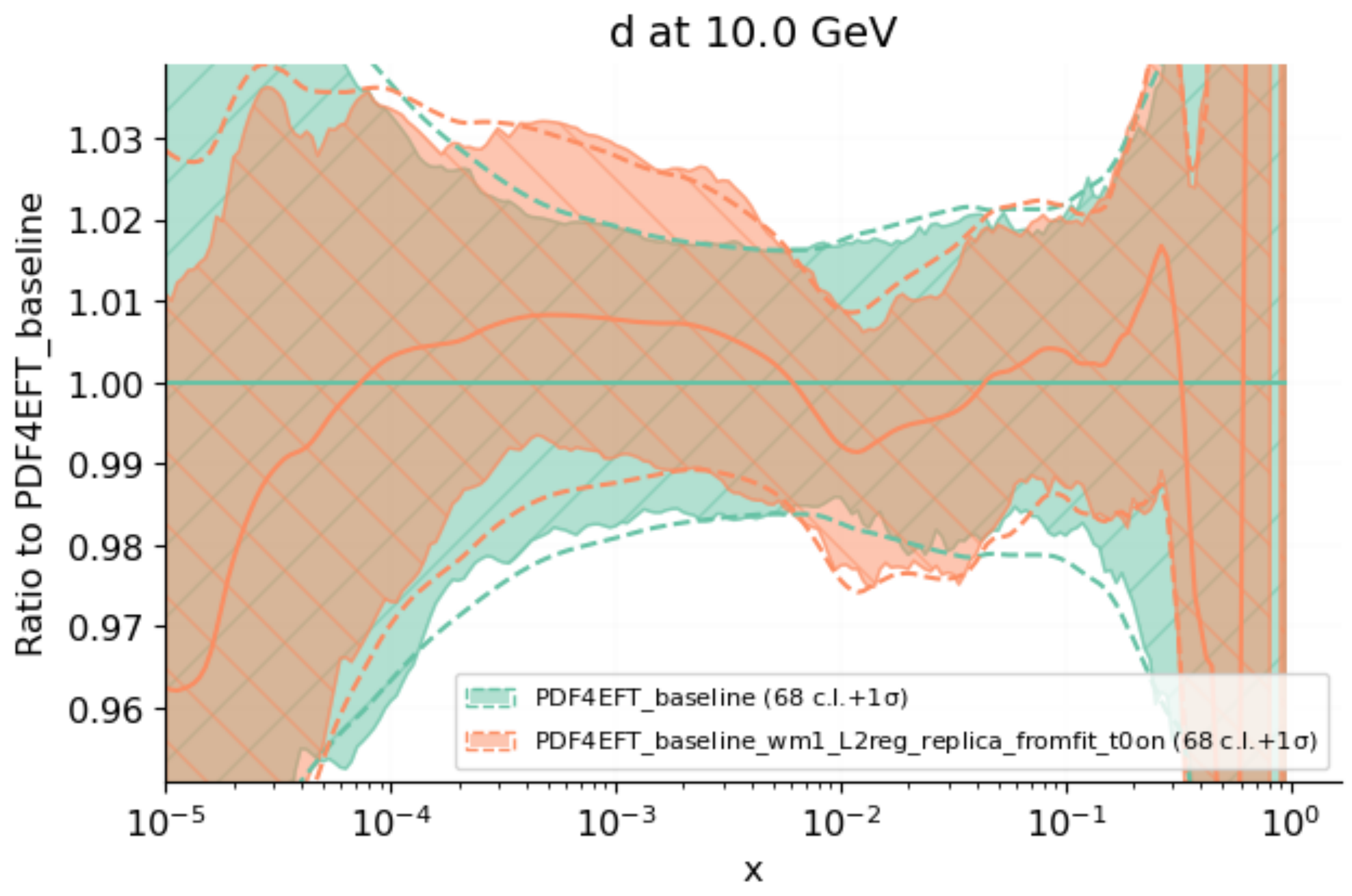
SECTION 4

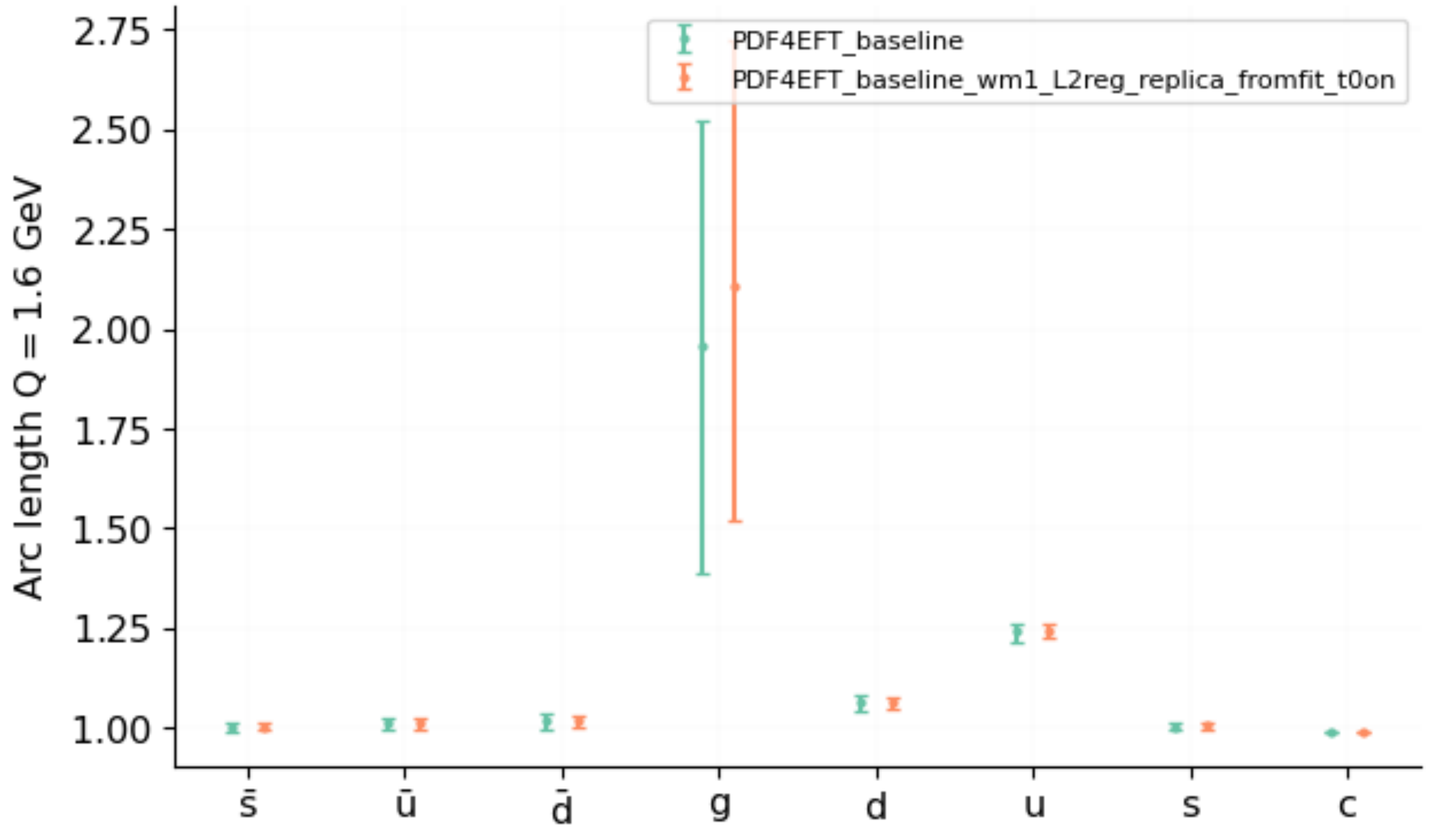
CURRENT RESULTS FROM THE METHODOLOGY

PDF-ONLY WEIGHT-MINIMISATION

- When SMEFT couplings are fixed to zero, weight-minimisation should **approximately reproduce the baseline PDFs**
- We see that this is indeed the case in both DIS-only and global scenarios: the PDFs change slightly and there is a small improvement in χ^2 reflecting the fact that adding some extra degrees of freedom can improve the fit





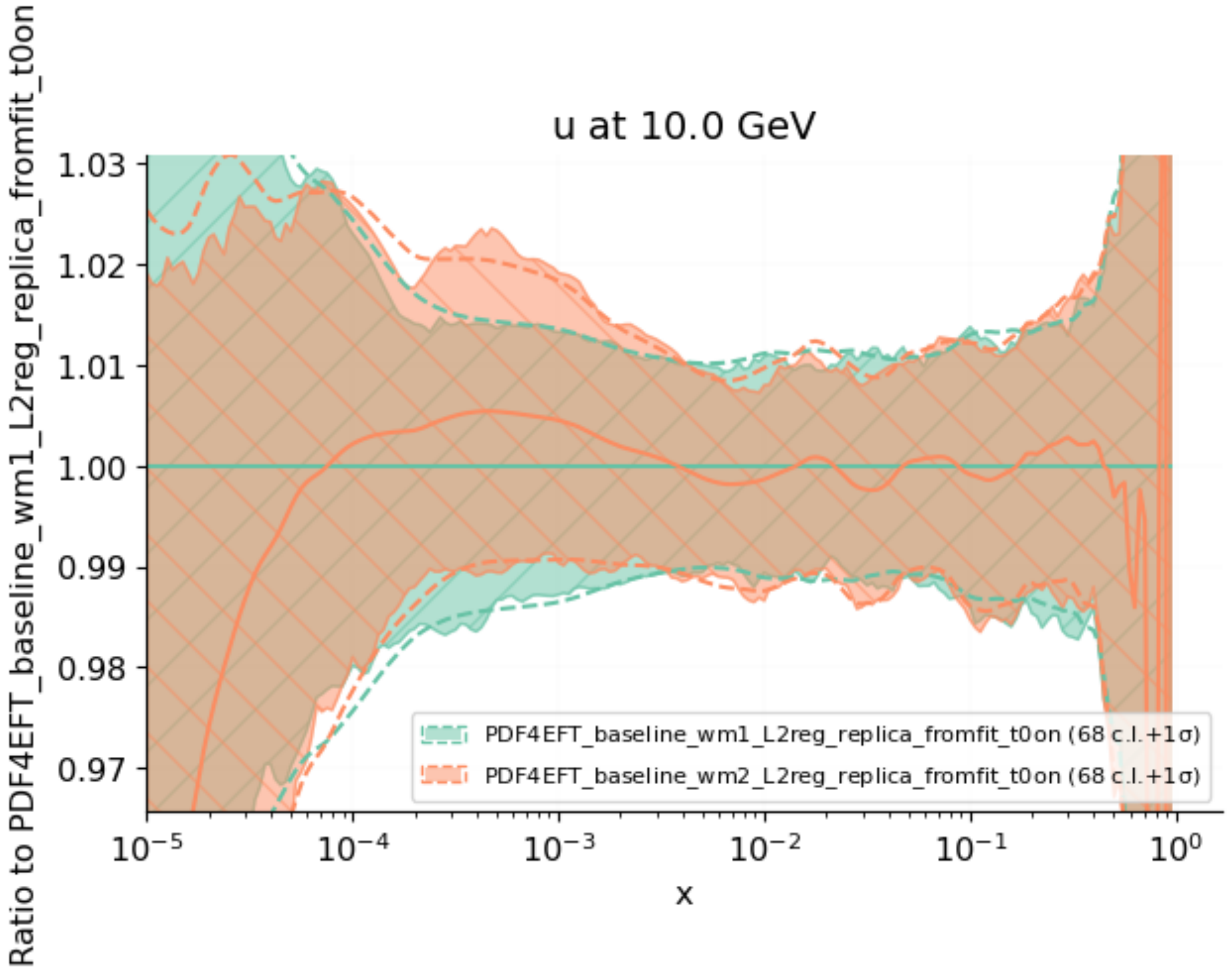


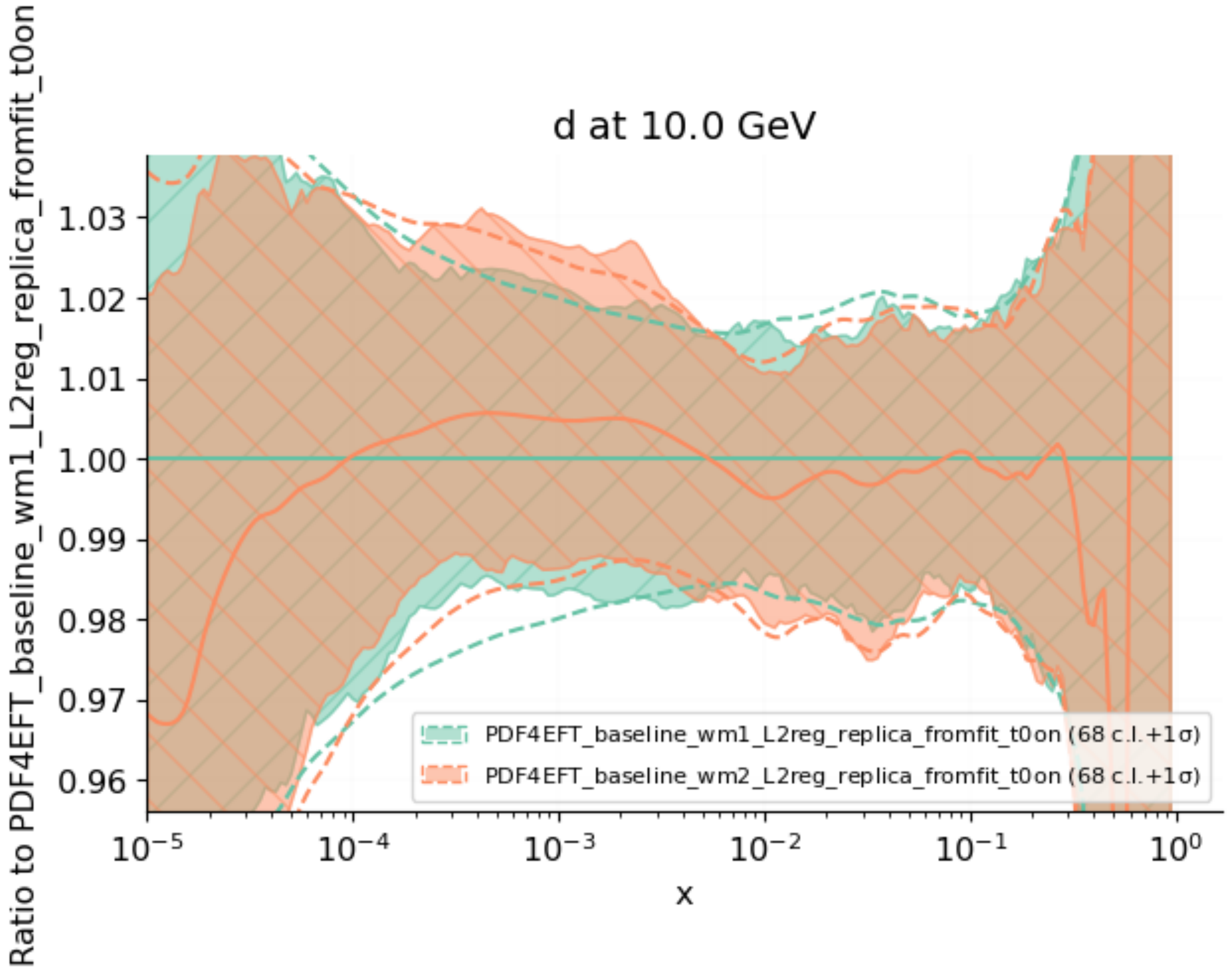
PDF-ONLY WEIGHT-MINIMISATION

- Improvement in χ^2 is possible, since we have effectively changed the architecture of the PDFs - we have **'stacked'** the neural network against a linear model
- We are interested in comparing our results to NNPDF 4.0 and will hopefully do so soon

PDF-ONLY WEIGHT-MINIMISATION

- Importantly, if we perform weight-minimisation twice, we see that the PDFs stay approximately the same on the second iteration
- Reflects the fact we have found the **optimal solution** (given our linear model)





PDF + SMEFT WEIGHT-MINIMISATION

- Results above suggest that we should take the following steps when doing PDF + SMEFT weight-minimisation:

  1. First apply weight-minimisation to baseline with SMEFT couplings fixed to zero. Call the resulting PDF set **WM1**.

  2. Perform weight-minimisation again on WM1 with SMEFT couplings fixed to zero. Operation is very close to identity, result is called **WM2**.

  3. Starting from **WM1** PDF set, perform weight-minimisation **including** the SMEFT sector. Deviation between the resulting PDF and **WM2** reflects the change in the PDFs due to simultaneous fit with SMEFT.

RESULTS FOR GLOBAL SCENARIOS

- In old high-mass DY paper, two SMEFT couplings we looked at were the **W** and **Y** parameters - these are combinations of four-fermion operators drawn from the Warsaw basis
- In old study, we fitted PDFs + W simultaneously, and also PDFs + Y simultaneously
- Same analysis with weight-minimisation methodology is as follows

PDF + W SIMULTANEOUS FIT

- 95% confidence intervals for W, from old paper:

	Fixed PDFs (no uncertainty)	Fixed PDFs (inc. uncertainty)	Simultaneous determination
$W \times 10^3$	[-5.5, 4.7]	[-6.8, 6.3]	[-6.4, 5.3]

- Change quantified by **broadening**: defined to be

$$\frac{\text{length SMEFT interval} - \text{length SM interval}}{\text{length SM interval}}$$

PDF + W SIMULTANEOUS FIT

- Broadening statistics from old paper:

	Broadening from fixed PDF (no uncertainty)	Broadening from fixed PDF (inc. uncertainty)
Broadening	15%	-11%

PDF + W SIMULTANEOUS FIT

- Comparison of corresponding results using weight-minimisation:

	Fixed PDFs (no uncertainty)	Fixed PDFs (inc. uncertainty)	Simultaneous determination
Old paper	[-5.5, 4.7]	[-6.8, 6.3]	[-6.4, 5.3]
Weight-minimisation	[-4.9, 4.5]	[-5.6, 5.1]	[-5.2, 5.0]

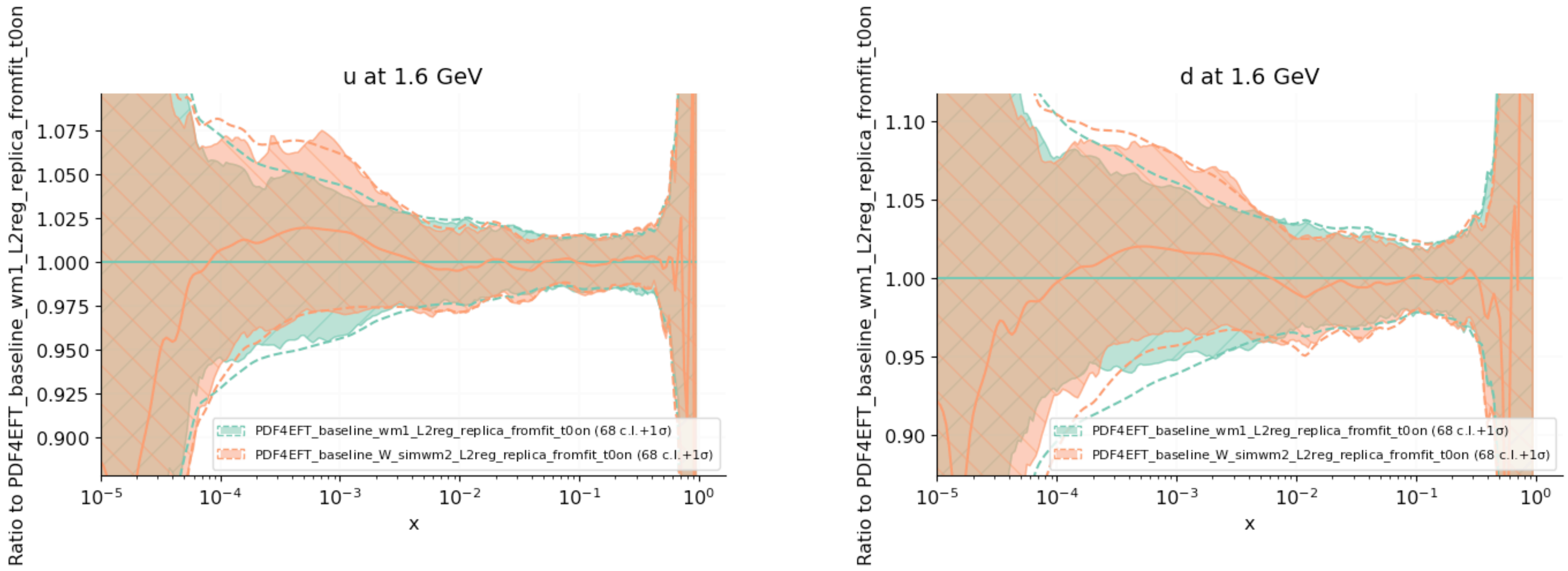
PDF + W SIMULTANEOUS FIT

- Comparison of corresponding results using weight-minimisation:

	Broadening from fixed PDF (no uncertainty)	Broadening from fixed PDF (inc. uncertainty)
Old paper broadening	15%	-11%
Weight-minimisation broadening	9.0%	-4.7%

PDF + W SIMULTANEOUS FIT

- Simultaneous PDFs (made starting from WM1) show **very small deviation** from WM1 PDFs



PDF + Y SIMULTANEOUS FIT

- Comparison of corresponding results using weight-minimisation:

	Fixed PDFs (no uncertainty)	Fixed PDFs (inc. uncertainty)	Simultaneous determination
Old paper	[-8.8, 9.2]	[-11, 12]	[-8.3, 12]
Weight-minimisation	[-7.6, 8.9]	[-8.8, 10]	[-8.1, 10]

PDF + Y SIMULTANEOUS FIT

- Comparison of corresponding results using weight-minimisation:

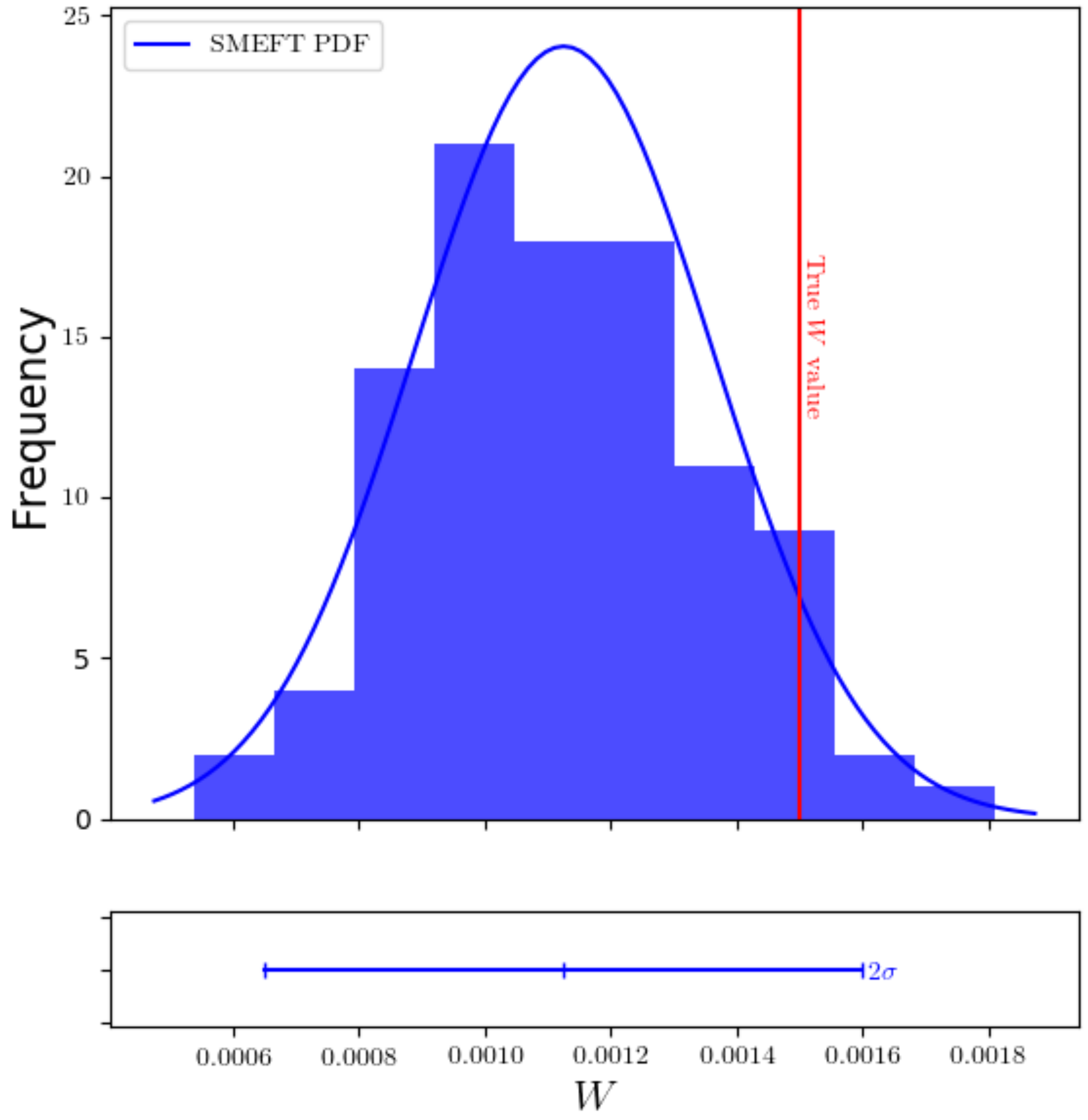
	Broadening from fixed PDF (no uncertainty)	Broadening from fixed PDF (inc. uncertainty)
Old paper broadening	12%	-13%
Weight-minimisation broadening	9.1%	-4.8%

CLOSURE TESTS: DETECTING NEW PHYSICS

- The third important benchmark of the method is that it can accurately detect New Physics when it is present.
- One way of testing this is to produce artificial data from a **fixed PDF set** with **SMEFT couplings fixed to certain non-zero values** chosen by the user.
- We then apply a SM NNPDF fit, followed by simultaneous WM to the resulting PDF fit, then we hope that WM can detect the SMEFT coupling.

CLOSURE TESTS: DETECTING NEW PHYSICS

- This has been checked in the case of the W-parameter, with NNPDF3.1 as the underlying law:



CONCLUSIONS

ADVANTAGES OF WEIGHT MINIMISATION

- **Efficient** - runs on a **laptop** (specifically Intel i5 dual-core processor), global fit takes around 12 hours
- **Can start from any Monte Carlo PDF set** - not just NNPDF
- **Extends to moderate/large numbers of SMEFT couplings** - theoretically no problem extending to more couplings
- **Could in principle be used for new data** - can use same 'linearisation of the deviation' method to add new data to an existing PDF fit efficiently (natural comparison with 'old reweighting')